

Non-Standard Neutrino Interactions at One Loop

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Abstract

Neutrino oscillation experiments are known to be sensitive to Non-Standard Interactions (NSIs). We extend the NSI formalism to include one-loop effects. We discuss universal effects induced by corrections to the tree level W exchange, as well as non-universal effects that can arise from scalar charged current interactions. We show how the parameters that can be extracted from the experiments are obtained from various loop amplitudes, which include vertex corrections, wave function renormalizations, mass corrections as well as box diagrams. As an illustrative example, we discuss NSIs at one loop in the Minimal Supersymmetric Standard Model (MSSM) with generic lepton flavor violating sources in the soft sector. We argue that the size of one-loop NSIs can be large enough to be probed in future neutrino oscillation experiments.

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I. INTRODUCTION

There are many experimental results confirming that neutrinos have masses and oscillate between different flavors [1]. It is plausible that neutrinos acquire small masses from some high scale physics via the see-saw mechanism [2]. If such high energy dynamics are the only source of lepton flavor violation (LFV), neutrino oscillations could remain their only observable effects.¹ It is possible, however, that extra sources of LFV are present at the weak scale. Such LFV sources could induce lepton number violating decays of charged leptons such as $\mu \rightarrow e\gamma$, $\mu \rightarrow eee$ and $\tau \rightarrow \mu\gamma$. Weak scale LFV interactions can also affect neutrino oscillation experiments. For instance, flavor violating Non-Standard Interactions (NSIs) can modify neutrino propagation in matter [5–10].

NSIs could also affect the production and detection processes generating “wrong flavor” neutrinos [11–13]. Consider for instance an appearance experiment where a neutrino is produced in association with a muon, and a tau is detected at the detector. The standard interpretation of such a result is that it is due to $\nu_\mu \rightarrow \nu_\tau$ oscillations. This interpretation is correct as long as the produced neutrino is orthogonal to the detected one. However, if there are NSIs that violate the flavor symmetry at the source, the produced neutrino, which we denote by $|\nu_\mu^s\rangle$ is not a simple flavor eigenstate. Analogously, in the presence of detector NSIs, the final state, which we denote as $|\nu_\tau^d\rangle$, is not a simple flavor eigenstate. If these states are not orthogonal, that is,

$$\langle \nu_\mu^s | \nu_\tau^d \rangle \equiv \varepsilon_{\mu\tau} \neq 0 , \quad (1)$$

tau appearance can occur without oscillation.

In order to describe the effect of NSIs on neutrino oscillation experiments, let us discuss first a simple case with only two generations and a mixing angle $\theta = \pi/4$. Consider also the case when the detector is at a distance L much smaller than one oscillation length,

$$x \equiv \frac{\Delta m^2 L}{4E} \ll 1 . \quad (2)$$

In this case the oscillation amplitude simply reads $\mathcal{A}_{\text{osc}}(\nu_\mu \rightarrow \nu_\tau) \approx ix$. When NSIs are present, they induce an extra contribution, $\mathcal{A}_{\text{NSI}}(\nu_\mu \rightarrow \nu_\tau) = \varepsilon_{\mu\tau}$. Thus, to leading order in x , the total appearance probability is given by the squared of the sum of amplitudes

$$\mathcal{P}(\nu_\mu \rightarrow \nu_\tau) \approx |\mathcal{A}_{\text{osc}} + \mathcal{A}_{\text{NSI}}|^2 \approx x^2 + |\varepsilon_{\mu\tau}|^2 + 2x \mathcal{I}m(\varepsilon_{\mu\tau}) . \quad (3)$$

The above simplified result has all the physics in it: the first term is the pure oscillation term, the second one is the x independent term that arises due to the NSIs, and the third

¹ A remarkable exception arises in the context of supersymmetric theories where such high scale dynamics could leave indelible footprints on the soft terms of the light sparticles via interactions not suppressed by inverse powers of the high scale [3, 4].

term is an interference term. Note that, when $x \gg \varepsilon_{\mu\tau}$, the probability to detect a new physics effect is *enhanced* by the interference term, which is linear in $\varepsilon_{\mu\tau}$, compared to the typical quadratic dependence of lepton flavor violating decays of charged leptons [12]. This fact triggered renewed interest at present neutrino facilities [14–19]. The typical bounds on production and detection NSIs are about $\varepsilon_{\mu\tau} \sim 10^{-2}$ [20, 21]. Higher sensitivities, of $\varepsilon_{\mu\tau} \lesssim 10^{-3}$ are within the reach of future neutrino experiments [22–30].

In this paper we study NSIs at the loop level, extending the formalism of [11, 12]. We present a general framework that allows one to extract in a consistent way the physical parameters $\varepsilon_{\alpha\beta}$ (with $\alpha, \beta = 1, 3$) which arise at the loop level either from corrections to the tree level W exchange diagram or from more general corrections, in particular from scalar charged currents. We show how the physical parameter, $\varepsilon_{\alpha\beta}$, can be obtained from the various loop amplitudes which include vertex corrections, wave function renormalizations, mass corrections as well as box diagrams.

In the case of universal corrections to the W exchange amplitudes, NSIs emerge at one loop because after the Electroweak Symmetry Breaking (EWSB) the kinetic terms and the W couplings are generally not universal in the same basis. Rotating to the mass basis for the charged leptons, the misalignment between vertex and wave functions induce NSIs. We show that the associated $\varepsilon_{\alpha\beta}$ are finite because the $SU(2)_L$ gauge symmetry protects them from possibly divergent contributions.

This paper is organized as follows. In Sec. II, we recall the standard formalism for NSIs. In Sec. III, we extend it to account for loop induced NSIs. In Sec. IV, we discuss NSIs at one loop in the R-parity conserving Minimal Supersymmetric Standard Model (MSSM) with generic LFV sources in the soft sector, leaving the details of the calculation to the appendix. Finally, we present our conclusions in Sec. V.

II. NOTATIONS AND FORMALISM

We start by setting our notations to follow Refs. [11, 12]. We first consider only tree level processes and later discuss one loop effects.

Neutrino mass eigenstates are denoted by $|\nu_i\rangle$, $i = 1, 2, 3$, while $|\nu_\alpha\rangle$ are the tree level weak interaction partners of the charged lepton mass eigenstates α^- , $\alpha = e, \mu, \tau$. These two bases are related by

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle . \quad (4)$$

where U is the leptonic mixing matrix, the so-called PMNS matrix [31].

We consider experiments where neutrinos are produced at the source in conjunction with incoming negative or outgoing positive charged leptons. Then, the neutrinos travel to the detector where they are detected by producing negative charged leptons. We consider

new physics in the production and detection processes assuming that these NSIs have the same Dirac structure as the SM interactions and thus the amplitudes add coherently. We parameterize the new interactions at the source and at the detector by two sets of effective four-fermion couplings,

$$(G_{\text{NP}}^s)_{\alpha\beta}, \quad (G_{\text{NP}}^d)_{\alpha\beta}, \quad (5)$$

where α is the charged lepton index and β is the flavor of the neutrino in the weak interaction basis. At tree level, the $SU(2)_L$ gauge symmetry implies that in the SM the four-fermion couplings are proportional to $G_F \delta_{\alpha\beta}$. New interactions, however, allow for non-diagonal and non-universal couplings.

Phenomenological constraints imply that the new interactions are suppressed with respect to the weak interactions. It is thus convenient to define small dimensionless quantities in the following way:

$$\epsilon_{\alpha\beta}^p \equiv \frac{(G_{\text{NP}}^p)_{\alpha\beta}}{\sqrt{|G_F + (G_{\text{NP}}^p)_{\alpha\alpha}|^2 + \sum_{\gamma \neq \alpha} |(G_{\text{NP}}^p)_{\alpha\gamma}|^2}} \quad p = s, d. \quad (6)$$

We denote by $|\nu_\alpha^s\rangle$ the neutrino states that is produced at the source, and by $|\nu_\alpha^d\rangle$ the neutrino state that is detected

$$|\nu_\alpha^p\rangle = \frac{G_F \delta_{\alpha\beta} + (G_{\text{NP}}^p)_{\alpha\beta}}{\sqrt{|G_F + (G_{\text{NP}}^p)_{\alpha\alpha}|^2 + \sum_{\gamma \neq \alpha} |(G_{\text{NP}}^p)_{\alpha\gamma}|^2}} |\nu_\beta\rangle \quad p = s, d. \quad (7)$$

At the leading order, we have

$$\epsilon_{\alpha\beta}^p = \frac{(G_{\text{NP}}^p)_{\alpha\beta}}{G_F}, \quad |\nu_\alpha^p\rangle = |\nu_\alpha\rangle + \epsilon_{\alpha\beta}^p |\nu_\beta\rangle. \quad (8)$$

The expression for the non-orthogonality parameter $\varepsilon_{\alpha\beta}$ at the leading order is given by

$$\varepsilon_{\alpha\beta} \equiv \langle \nu_\alpha^s | \nu_\beta^d \rangle = \begin{cases} 1 + \mathcal{O}(\epsilon^2) & \alpha = \beta \\ \epsilon_{\alpha\beta}^{s*} + \epsilon_{\beta\alpha}^d + \mathcal{O}(\epsilon^2) & \alpha \neq \beta \end{cases}, \quad (9)$$

where by $\mathcal{O}(\epsilon^2)$ we refer to effects that are quadratic in ϵ^s or ϵ^d . Note that in the SM the non-orthogonality parameter vanishes, $\varepsilon_{\alpha\neq\beta} = 0$.

For simplicity, we consider now a two generation model where the production process is associated with a muon and the detection with a tau. We calculate the following expression for the transition probability

$$P_{\mu\tau} = |\langle \nu_\tau^d | \nu_\mu^s(t) \rangle|^2, \quad (10)$$

where $\nu_\mu^s(t)$ is the time-evolved state that was purely ν_μ^s at time $t = 0$. Using an explicit parameterization of the neutrino mixing matrix

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (11)$$

and keeping terms up to leading order in ε we get

$$P_{\mu\tau} = \sin^2 x \{ \sin^2 2\theta + \mathcal{R}e(\epsilon_{\tau\mu}^d - \epsilon_{\mu\tau}^s) \sin 4\theta \} + \sin 2x \mathcal{I}m(\varepsilon_{\mu\tau}) \sin 2\theta, \quad (12)$$

where

$$\Delta m_{ij}^2 \equiv m_i^2 - m_j^2, \quad x_{ij} \equiv \frac{\Delta_{ij} L}{4E}, \quad x = x_{12}, \quad (13)$$

m_i are the neutrino masses, E is the neutrino energy and L is the distance between the source and the detector.

A few remarks are in order regarding Eq. (12):

- We keep only terms up to $O(\varepsilon)$. This is the reason why there is no effect at $x = 0$.
- The different x dependence of the various terms is important as it can be used to distinguish them experimentally.
- The interference term can be very important when $x \gg \varepsilon$.
- The interference term depends on the imaginary part of the NSIs, that is, it requires CP violation.
- NSIs also affect the term proportional to $\sin^2 x$. Yet, within one experiment this change is absorbed into the definition of θ and cannot be distinguished experimentally.
- In many cases the NSIs are closely related to lepton flavor violating charged lepton decays. However, they have a different dependence on ε . Neutrino oscillation experiments are linear in ε (if $x \gg \varepsilon$) while decays like $\tau \rightarrow \mu e^+ e^-$ are quadratic. This makes neutrino oscillation experiments competitive in sensitivity.
- With three generations the result is more complicated and can be found in [12].

Before concluding this section, we remark on the case of heavy neutrinos. For instance, consider the case of k heavy singlet neutrinos. The mixing matrix is $3 \times (3 + k)$ and the 3×3 mixing matrix for the light neutrinos is not unitary anymore. In this case we have [32]

$$\varepsilon_{\alpha\beta} = \sum_h U_{h\alpha} U_{h\beta}^*. \quad (14)$$

The point is that using neutrino oscillation experiments we can measure ε , and claim detection of some new physics, but we can not disentangle the underlying mechanism which generates it.

III. ONE LOOP NSI

In this section, we extend the NSI formalism of Refs. [11, 12] to include one-loop effects. In particular, we consider universal NSIs from correction to the tree level W interaction and non-universal effects due to box diagrams and scalar charged currents.

A. Correction to the W exchange amplitude

At tree level, gauge invariance guarantees universality of the W interactions. This universality is kept to all orders for an exact symmetry. For a broken symmetry, however, universality is lost beyond tree level. In the following, we show that one-loop effects make the W couplings and the kinetic terms of the fermions non-universal. In particular, we explain why, in general, in the basis where the kinetic term are canonical, the W interactions are not flavor diagonal.

In the following, we neglect neutrino masses since they give subleading effects to the NSIs, as we will discuss later. Similarly, we do not consider other possible non-universal flavor-diagonal NSIs. They can be there but they are assumed to be small and, to leading order, we can just add them to the effect we are considering here.

On general grounds, one-particle irreducible one-loop effects include the self energy diagrams for the charged leptons and the neutrinos, and the corrections to the W vertex as well. These one-loop diagrams modify the kinetic and mass terms for fermions and the W vertex by factors Z_L^ν , $Z_{L,R}^\ell$, and η_m^ℓ defined as follows:

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & \bar{\ell}_{jL} (Z_L^\ell)^{ji} i \not{\partial} \ell_{iL} + \bar{\ell}_{jR} (Z_R^\ell)^{ji} i \not{\partial} \ell_{iR} + \bar{\nu}_{jL} (Z_L^\nu)^{ji} i \not{\partial} \nu_{iL} \\ & - \bar{\ell}_{jR} (m_\ell^\circ + \eta_m^\ell)^{ji} \ell_{iL} - \bar{\ell}_{jL} (m_\ell^{\circ\dagger} + \eta_m^{\ell\dagger})^{ji} \ell_{iR} \\ & - \frac{g}{\sqrt{2}} W_\mu^- \bar{\ell}_{jL} \gamma^\mu (Z_L^W)^{ji} \nu_{iL} + \text{H.c.} \quad i, j = 1, 2, 3, \end{aligned} \quad (15)$$

where the symbol “ \circ ” refers to “bare” or unrenormalized mass matrices and Z^a are matrices defined as

$$(Z^a)_{ij} = \delta_{ij} + (\eta^a)_{ij} \quad a = \nu, \ell, W. \quad (16)$$

Note that Hermiticity of the effective Lagrangian ensures that $(\eta^{\ell,\nu})^\dagger = \eta^{\ell,\nu}$ while η^W may not be Hermitian. While at tree level $\eta^a = 0$, lepton flavor violating loop corrections introduce non-zero η 's. In general, the off-diagonal elements of η^a are finite, while the diagonal terms diverge. Of course, the physics is finite and we discuss how the divergences cancel below. We define

$$\hat{Z}_L^{\ell,\nu} = L_{\ell,\nu}^\dagger Z_L^{\ell,\nu} L_{\ell,\nu}, \quad \hat{Z}_R^\ell = R_\ell^\dagger Z_R^\ell R_\ell, \quad (17)$$

such that $\hat{Z}_{L,R}^a$ are diagonal matrices of positive elements, and L_ℓ and R_ℓ are unitary matrices. We rescale the lepton fields to make their kinetic terms canonical

$$\nu_L \rightarrow L_\nu \left(\hat{Z}_L^\nu \right)^{-\frac{1}{2}} \nu_L, \quad \ell_L \rightarrow L_\ell \left(\hat{Z}_L^\ell \right)^{-\frac{1}{2}} \ell_L, \quad \ell_R \rightarrow R_\ell \left(\hat{Z}_R^\ell \right)^{-\frac{1}{2}} \ell_R. \quad (18)$$

where $(\hat{Z}^a)^{-1/2}$ is shorthand for the diagonal matrix of element $(\hat{Z}^a)_{ii}^{-1/2}$. The charged lepton mass terms become

$$\mathcal{L}_{\text{mass}} = -\bar{\ell}_{jR} \left(\hat{Z}_R^\ell \right)^{-\frac{1}{2}} R_\ell^\dagger \left(m_\ell^\circ + \eta_m^\ell \right) L_\ell \left(\hat{Z}_L^\ell \right)^{-\frac{1}{2}} \ell_{iL} + \text{h.c.} \quad (19)$$

The mass terms (19) can be diagonalized by two independent rotations

$$\ell_L \rightarrow L_m \ell_L, \quad \ell_R \rightarrow R_m \ell_R, \quad (20)$$

where L_m and R_m are unitary matrices. We obtain

$$\left[R_m^\dagger \left(\hat{Z}_R^\ell \right)^{-\frac{1}{2}} R_\ell^\dagger \left(m_\ell^\circ + \eta_m^\ell \right) L_\ell \left(\hat{Z}_L^\ell \right)^{-\frac{1}{2}} L_m \right]^{ij} = (m_\ell)_{ij}, \quad (21)$$

where m_ℓ is diagonal. After performing the rescaling (18) and the field rotation (20), the kinetic terms are canonical and the charged lepton mass matrix is diagonal, whereas the interaction terms are not diagonal. For later convenience, we rotate the neutrino fields as $\nu_L \rightarrow L_m \nu_L$, in order to keep them as much aligned as possible with their charged partners. Note that this choice is allowed because when we study NSIs we can neglect neutrino masses. As a result, the W -boson vertices become

$$\mathcal{L}_{int} = -\frac{g_2}{\sqrt{2}} W_\mu^- \bar{\ell}_L \gamma^\mu Z \nu_L - \frac{g_2}{\sqrt{2}} W_\mu^+ \bar{\nu}_L \gamma^\mu Z^\dagger \ell_L, \quad (22)$$

where

$$Z = L_m^\dagger \left(\hat{Z}_L^\ell \right)^{-\frac{1}{2}} L_\ell^\dagger Z_L^W L_\nu \left(\hat{Z}_L^\nu \right)^{-\frac{1}{2}} L_m, \quad (23)$$

Let us stress that eq. (23) is valid to all orders in perturbation theory. Finally, the relation to the physical parameter $\varepsilon_{\alpha\beta}^W$ can be derived from eq. (7) and is given by

$$\varepsilon_{\alpha\beta}^W = \langle \nu_\alpha^s | \nu_\beta^d \rangle = \frac{(ZZ^\dagger)_{\beta\alpha}}{\sqrt{(ZZ^\dagger)_{\alpha\alpha}(ZZ^\dagger)_{\beta\beta}}}. \quad (24)$$

This formula has a simple interpretation. $Z_{\alpha\beta}/\sqrt{(ZZ^\dagger)_{\alpha\alpha}} |\nu_\beta\rangle$ is the normalized state produced at the source, while its conjugate is the one detected.

We proceed to find the leading order expressions for $Z_{\alpha\beta}$ and $\varepsilon_{\alpha\beta}^W$. That is, we will work to one loop level. In this case, we can identify the off-diagonal terms of Z with the NSIs at the source and the detector, and therefore we find

$$Z_{\alpha\beta} = \epsilon_{\alpha\beta}^{s*} = \epsilon_{\alpha\beta}^{d*}, \quad \alpha \neq \beta. \quad (25)$$

For the physical parameters we then get

$$\varepsilon_{\alpha\beta}^W = Z_{\alpha\beta} + Z_{\beta\alpha}^*, \quad \alpha \neq \beta. \quad (26)$$

The same result can be obtained directly from eq. (24). Note that $ZZ^\dagger = \delta_{\alpha\beta}$ when Z is unitary. When the deviation from unitarity is small, $ZZ^\dagger = 1 + \varepsilon^W$, we recover the previous result. See also eq. (33) below.

At one loop, the transformations for the lepton fields of eq. (18) read

$$\nu_L \rightarrow \left(1 - \frac{1}{2} \eta_L^\nu \right) \nu_L, \quad \ell_L \rightarrow \left(1 - \frac{1}{2} \eta_L^\ell \right) \ell_L, \quad \ell_R \rightarrow \left(1 - \frac{1}{2} \eta_R^\ell \right) \ell_R. \quad (27)$$

Similarly, to one loop accuracy, the transformations of eq. (20) read

$$\ell_L \rightarrow (1 + \delta L_m) \ell_L, \quad \ell_R \rightarrow (1 + \delta R_m) \ell_R. \quad (28)$$

The unitarity of L_m and R_m implies that

$$\delta L_m^\dagger = -\delta L_m, \quad \delta R_m^\dagger = -\delta R_m. \quad (29)$$

In this approximation, Z is given by

$$Z = 1 + \eta_L^W - \frac{\eta_L^\ell + \eta_L^\nu}{2}, \quad (30)$$

where η^ν , η^ℓ , and η^W have to be evaluated at one loop accuracy. Eq. (30) shows that, at this level, the dependence of Z on L_m drops completely out (its dependence would be reintroduced at two loop). Then we learn that at leading order the non-orthogonality parameter between the source and detector neutrinos is given by (26) and reads

$$\varepsilon_{\alpha\beta}^W = \varepsilon_{\alpha\beta}^{s*} + \varepsilon_{\beta\alpha}^d = \left(\eta_L^{W\dagger} + \eta_L^W - \eta_L^\ell - \eta_L^\nu \right)_{\alpha\beta}, \quad (31)$$

where we used the fact that η^ℓ and η^ν are Hermitian.

Eqs. (23), (26), and (31) are the main results of this section. A few remarks are in order when inspecting them:

1. For any given model, eqs. (23), (26) show how to extract the physical NSI effects out of the calculations of the vertex corrections and the self energies encoded in the rotation mass matrices L_ν , L_ℓ and L_m . However, eq. (31) demonstrates that, at one loop accuracy, all we need to do is to calculate η^ν , η^ℓ , and η^W , since L_m starts contributing only at two loop.
2. We note that $\varepsilon_{\alpha\beta}^W$ is finite. While each of the diagonal terms in η^a may diverge, the combination is finite because of the $SU(2)_L$ gauge symmetry. This can be seen by the fact that the UV properties are insensitive to EWSB. Thus, the divergent part of η^a is real, flavor universal and independent of a . Therefore, the divergent part of $(\eta_L^{W\dagger} + \eta_L^W - \eta_L^\ell - \eta_L^\nu)_{\alpha\alpha}$, as well as its renormalization scale dependence, cancel. In contrast, the flavor off-diagonal elements of η_L^W and $\eta_L^{\nu,\ell}$ are singularly finite and scale independent, as a result of the GIM mechanism. We stress also that when the electroweak symmetry is restored, $v_{EW} \rightarrow 0$, then the finite parts are flavor universal and $\varepsilon_{\alpha\beta}^p \rightarrow 0$. Both results are illustrated in a concrete example below where the η^a are explicitly calculated.
3. Considering the CP-conjugated process, we obtain

$$\mathcal{A}_{NSI}^{CP} = \varepsilon_{\beta\alpha}^W = \varepsilon_{\alpha\beta}^{W*}. \quad (32)$$

So, CP is violated when the η^a have non trivial imaginary parts.

4. Eq. (31) admits an interpretation in terms of the scattering amplitudes $\mathcal{M}_{\alpha\beta}$. The idea is to think about neutrinos as invisible intermediate states, and sum over all of them in the propagation process from the source to the detector. Considering a source neutrino associated with a charged lepton α and a detection that is done by a charged lepton β , the scattering amplitude is given by

$$\mathcal{M}_{\alpha\beta} \propto (ZZ^\dagger)_{\beta\alpha} = \delta_{\beta\alpha} + \left(\eta_L^W + \eta_L^{W\dagger} - \eta_L^\nu - \eta_L^\ell \right)_{\beta\alpha} = \delta_{\alpha\beta} + \varepsilon_{\beta\alpha}^W. \quad (33)$$

where in the second step we expanded in $\eta_L^{\nu,l,W}$. Thus, we learn that the non-universal part of the amplitude is just $\varepsilon_{\beta\alpha}^W$.

B. Non-universal effects

So far, we have considered only NSIs induced by the universal corrections to the W vertex and self energy diagrams. These effects are independent of the production or detection processes as long as these processes are mediated by W exchange. We now move to discuss other loop effects that are not universal, that is, that may be different for different production or detection processes.

As mentioned before, the universal effects are $SU(2)$ breaking effects and therefore suppressed like M_W^2/M_{NP}^2 . In the effective field theory language, this would correspond to the effects induced, after the EWSB, by gauge invariant dimension-six operators like $(\bar{L}_L \tau^a \gamma^\mu L_L)(H^\dagger \tau^a D_\mu H)$, where L_L (H) stands for the lepton (Higgs) doublet, τ^a are either the identity or the $SU(2)$ generators and D_μ is the covariant derivative.

It is then clear that contributions arising from dimension six four-fermion operators, also suppressed by M_W^2/M_{NP}^2 , must be consistently included along with the W -penguin contributions. Therefore, for any given New Physics (NP) model we can write the following expression for the physical $\varepsilon_{\alpha\beta}$ parameter

$$\varepsilon_{\alpha\beta} = \varepsilon_{\alpha\beta}^W + (\epsilon_{\alpha\beta}^{s*})^{dim-6} + (\epsilon_{\beta\alpha}^d)^{dim-6}. \quad (34)$$

where $\varepsilon_{\alpha\beta}^W$ has been already defined in eq. (31) and we have assumed both matter effects and higher order operators (with $\dim > 6$) to be negligible.

In this work we are interested in NSIs with the same Dirac structure as the SM interactions. The reason is that they maximally exploit the interference between SM and NP amplitudes. In particular, focusing on realistic production and detection processes like $\mu \rightarrow e \nu_e \bar{\nu}_\alpha$ and $P \rightarrow \mu \nu_\alpha$ (with $P = \pi, K$), the relevant dimension six operators are the following

$$\frac{4G_F}{\sqrt{2}} (\delta_{\mu\alpha} + (\epsilon_{\mu\alpha}^s)^{dim-6}) (\bar{\nu}_\alpha \gamma^\lambda P_L \mu) (\bar{e} \gamma_\lambda P_L \nu_e), \quad (35)$$

$$\frac{4G_F}{\sqrt{2}} (\delta_{\mu\alpha} + (\epsilon_{\mu\alpha}^d)^{dim-6}) (\bar{u} \gamma^\lambda P_L d) (\bar{\mu} \gamma_\lambda P_L \nu_\alpha), \quad (36)$$

where $(\epsilon_{\mu\alpha}^{s,d})^{dim-6}$ stand for the loop-induced corrections.

As we will discuss in the next section, in the context of supersymmetry $\epsilon_{\mu\alpha}^{dim-6}$ might be induced either by means of gaugino/sfermion boxes or through the tree level exchange of charged Higgs with loop induced flavor changing couplings $H\ell\nu$.

C. Matter effects

So far, we have discussed only NSIs induced by the charged currents. However, we would like to emphasize here that there exist also NSIs in matter via neutral currents, even for negligible SM matter effects.

Indeed, starting from the SM neutral current interactions

$$\mathcal{L}_{\text{eff}} = -\frac{g}{2c_W} Z_\mu \bar{\nu}_{jL} \gamma^\mu (Z_L^Z)^{ji} \nu_{iL} + \text{H.c.} \quad i, j = 1, 2, 3, \quad (37)$$

where hereafter $c_W = \cos \theta_W$ and $s_W = \sin \theta_W$, after performing the rescaling (18) and the field rotation (20), the Z -boson vertex with neutrinos is given by

$$Z = L_m^\dagger \left(\hat{Z}_L^\nu \right)^{-\frac{1}{2}} L_\nu^\dagger Z_L^Z L_\nu \left(\hat{Z}_L^\nu \right)^{-\frac{1}{2}} L_m, \quad (38)$$

where, again, we rotated the neutrino fields as $\nu_L \rightarrow L_m \nu_L$. At one loop accuracy, eq. (38) becomes

$$Z = 1 + \eta_L^Z - \eta_L^\nu. \quad (39)$$

We stress that also NSIs induced by the Z -penguin are $SU(2)$ breaking effects and all the considerations made for W -penguin induced NSIs apply also here. Therefore, besides Z -penguin effects, we have to include dimension six four-fermion operators. The relevant neutral interactions are

$$\sqrt{2} G_F \left[\varepsilon_{\mu\alpha}^Z \left(I_{3L}^f - 2Q_f s_W^2 \right) + (\varepsilon_{\mu\alpha}^{m,f})^{dim-6} \right] (\bar{\nu}_\alpha \gamma^\lambda P_L \nu_\mu) (\bar{f} \gamma_\lambda f), \quad (40)$$

where f stands for the fermions in the matter. For normal matter, f could be electrons, protons and neutrons $f = e, p, n$, Q_f is the electric charge of f and I_{3L}^f is the third component of weak isospin of the left-chiral projection of f .

Since we can identify $\varepsilon_{\mu\alpha}^Z = Z_{\mu\alpha}$, the total physical parameter in the matter $\varepsilon_{\mu\tau}^{m,f}$ reads

$$\varepsilon_{\mu\tau}^{m,f} = \left(I_{3L}^f - 2Q_f s_W^2 \right) (\eta_L^Z - \eta_L^\nu)_{\mu\tau} + (\varepsilon_{\mu\tau}^{m,f})^{dim-6}. \quad (41)$$

When matter effects are included, the transition probability $P_{\mu\tau}$ of Eq. (12) becomes

$$\begin{aligned} P_{\mu\tau} \simeq & x \sin 2\theta \left[x \sin 2\theta - 2L \sum_f A^f \mathcal{R}e(\varepsilon_{\mu\tau}^{m,f}) \right] \\ & + x^2 \mathcal{R}e(\epsilon_{\tau\mu}^d - \epsilon_{\mu\tau}^s) \sin 4\theta + 2x \mathcal{I}m(\varepsilon_{\mu\tau}) \sin 2\theta, \end{aligned} \quad (42)$$

where $A^f = \sqrt{2}G_F n_f$ and we have assumed $x \ll 1$ and constant fermion densities n_f .

A close look to Eq. (42) shows that the interference term between the SM and non-SM matter effects ($\varepsilon_{\mu\tau}^{m,f}$) depends only on the real part of $\varepsilon_{\mu\tau}^{m,f}$. By contrast, for NSIs at the production or detection processes ($\varepsilon_{\mu\tau}^{s,d}$), the interference term depends only on the imaginary part of the NSIs.

The situation changes when considering the transition probabilities $P_{e\mu}$ and $P_{e\tau}$ which involve the electron neutrino. In these cases, there are interference terms, driven by charged current SM-effects, which are also sensitive to the real parts of $\varepsilon^{s,d}$ [12].

D. Scalar charged current

Many UV completions of the SM contain an extended Higgs sector, for example, the MSSM. On general grounds, the presence of at least two Higgs doublets leads to a misalignment between the fermion mass matrices and the Yukawa couplings. As a result, Higgs mediated FCNC processes are induced already at tree level resulting in large effects, unless a flavor protection mechanism is at work. Indeed, the Natural Flavor Conservation (NFC) hypothesis was introduced to deal with this flavor problem. However, even if NFC holds at the tree level, this hypothesis is spoiled by quantum corrections [33]. For instance, if NFC arises as a result of a continuous PQ symmetry, the breaking at the quantum level of such a symmetry (as it is required in order to prevent the appearance of a massless Goldstone boson) would reintroduce FCNC effects [33].

This is the case of supersymmetry where the holomorphy of the superpotential implies a type-II structure of the Higgs potential at the tree level. Yet, the presence of a non-vanishing μ -term (such that $\mu H_u H_d$) induces, after SUSY breaking, non-holomorphic Yukawa couplings for fermions (such as $\bar{Q}_L d_R H_u^\dagger$) [34] and therefore Higgs-mediated flavor violation is unavoidable [35].

Bearing in mind the above considerations, in the following we perform a model independent analysis of NSIs arising from loop-induced scalar charged currents.

The charged Higgs H^\pm couplings with leptons are described by the following effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \bar{\nu}_{jL} (y_\ell^\circ + \eta^H)^{ji} \ell_{iR} H^+ + \text{H.c.} \quad (43)$$

where

$$y_\ell^\circ = \frac{g_2}{\sqrt{2}M_W} m_\ell^\circ \tan \beta \quad (44)$$

and we recall that “ \circ ” refers to unrenormalized quantities.

The transformations of eqs. (18), (20) lead to the following effective Lagrangian valid to

all orders in perturbation theory

$$\mathcal{L}_{\text{eff}} = \bar{\nu}_{jL} \left[L_m^\dagger \left(\hat{Z}_L^\nu \right)^{-\frac{1}{2}} L_\nu^\dagger \left(y_\ell^\circ + \eta^H \right) R_\ell \left(\hat{Z}_R^\ell \right)^{-\frac{1}{2}} R_m \right]^{ji} \ell_{iR} H^+ + \text{H.c.} . \quad (45)$$

In order to find the leading expansion for the above effective couplings, we proceed as follows. We first rescale the lepton fields at one loop level, see eq. (18), so that the charged lepton mass terms become

$$\mathcal{L}_{\text{mass}} = -\bar{\ell}_{jR} (m_\ell^\circ + \delta m_\ell)^{ji} \ell_{iL} + \text{h.c.} , \quad (46)$$

where

$$\delta m_\ell \equiv \eta_m^\ell - \frac{1}{2} \eta_R^\ell m_\ell^\circ - \frac{1}{2} m_\ell^\circ \eta_L^\ell . \quad (47)$$

Then, we make use of the one loop expansions for the matrices L_m and R_m of eq. (28) leading to

$$\left[m_\ell^\circ + \delta m_\ell - \delta R_m m_\ell^\circ + m_\ell^\circ \delta L_m \right]^{ji} = m_\ell^{ji} , \quad (48)$$

where m_ℓ is diagonal and we have consistently retained only the leading one-loop terms. At this level, the unitarity condition of eq. (29) ensures that $\text{Re}(\delta L_m^{jj}) = \text{Re}(\delta R_m^{jj}) = 0$ and the corrected mass eigenvalues are given by

$$m_{\ell_j} = m_{\ell_j}^\circ + \text{Re}(\delta m_\ell)_{jj} , \quad (49)$$

while the condition of reality for the masses implies

$$\text{Im} \delta R_m^{jj} - \text{Im} \delta L_m^{jj} = \frac{\text{Im}(\delta m_\ell)_{jj}}{m_{\ell_j}} . \quad (50)$$

Finally, δL_m and δR_m are determined by

$$\delta L_m^{ji} = \frac{m_{\ell_j}(\delta m_\ell)^{ji} + (\delta m_\ell^\dagger)^{ji} m_{\ell_i}}{m_{\ell_i}^2 - m_{\ell_j}^2} \quad j \neq i , \quad (51)$$

$$\delta R_m^{ji} = \frac{m_{\ell_j}(\delta m_\ell^\dagger)^{ji} + (\delta m_\ell)^{ji} m_{\ell_i}}{m_{\ell_i}^2 - m_{\ell_j}^2} \quad j \neq i , \quad (52)$$

where $m_{\ell_i} = (m_\ell)_{ii}$, that is, it is the i th eigenvalue of m_ℓ . We are ready now to expand eq. (45) up to one loop. By making use of the eqs. (47), (49), and (50), we obtain the following flavor conserving couplings

$$\mathcal{L}_{\text{eff}}^{H^+} = \bar{\nu}_{iL} \left[y_{\ell_i} \left(1 + \frac{1}{2} \eta_L^\ell - \frac{1}{2} \eta_L^\nu - \frac{\eta_m^{\ell\dagger}}{m_{\ell_i}} \right) + \eta^H \right]^{ii} \ell_{iR} H^+ + \text{H.c.} \quad (53)$$

where

$$y_{\ell_i} = \frac{g_2 m_{\ell_i}}{\sqrt{2} M_W} \tan \beta. \quad (54)$$

Again, while each term in eq. (53) is in general divergent and renormalization scale dependent, their sum is finite and scale independent.

For the flavor violating charged Higgs couplings, we find the one loop expression

$$\mathcal{L}_{\text{eff}}^{H^+} = \bar{\nu}_{jL} \left[-y_{\ell_i} \delta L_m + y_{\ell_j} \delta R_m - \frac{1}{2} y_{\ell_i} \eta_L^\nu - \frac{1}{2} y_{\ell_j} \eta_R^\ell + \eta^H \right]^{ji} \ell_{iR} H^+ + \text{H.c.}, \quad (55)$$

and each term in eq. (55) is finite and scale independent thanks to the GIM mechanism. Notice that, in contrast to the case of NSIs at the W -boson vertex, we are now sensitive to the rotation mass matrices L_m and R_m already at the one loop level. As we will discuss in the next section, within the MSSM the rotations δL_m and δR_m actually provide the dominant effects to the flavor changing couplings.

Let us consider now the case of $j = 3$, that is relevant for a tau neutrino production. In such a case, the one loop expansions for δL_m and δR_m of eqs. (51) and (52) take the form

$$(\delta R_m)^{3i} \simeq \left[-\frac{\eta_m^{\ell\dagger}}{m_\tau} + \frac{1}{2} \eta_R^\ell - \frac{m_\mu}{m_\tau} \frac{\eta_m^{\ell\dagger}}{m_\tau} + \frac{m_\mu}{m_\tau} \eta_L^\ell \right]^{3i}, \quad (56)$$

$$(\delta L_m)^{3i} \simeq \left[-\frac{\eta_m^\ell}{m_\tau} + \frac{1}{2} \eta_L^\ell \right]^{3i}. \quad (57)$$

Finally, inserting the above expressions for δL_m and δR_m in eq. (55), we find

$$\mathcal{L}_{\text{eff}}^{H^+} = \bar{\nu}_{\tau L} Z_H^{3i} \ell_{iR} H^+ + \text{H.c.}, \quad (58)$$

where

$$Z_H^{3i} = \left[\frac{1}{2} y_{\ell_i} \eta_L^\ell - \frac{1}{2} y_{\ell_i} \eta_L^\nu - y_\tau \frac{\eta_m^{\ell\dagger}}{m_\tau} + \eta^H \right]^{3i}. \quad (59)$$

Let us remark that NSI effects driven by charged scalar currents are expected to be particularly relevant for the neutrino production via charged meson decays. In fact, whenever the relevant Yukawa couplings are proportional to the fermion masses, only processes like $P \rightarrow \ell \nu$ (with $P = K, \pi$), which are helicity suppressed in the SM, might receive large contributions. Other production processes like $\mu \rightarrow e \nu \bar{\nu}$ or detection cross-sections are expected not to be significantly affected by such charged scalar currents. As a result, we now have $\epsilon_{\alpha\beta}^s \gg \epsilon_{\alpha\beta}^d$ and therefore $\varepsilon_{\mu\tau} \approx \epsilon_{\mu\tau}^{s*}$. This is in contrast to the case with dominant NSIs at the W -boson vertex where, as we already discussed, it turns out that $\epsilon_{\alpha\beta}^s = \epsilon_{\alpha\beta}^d$.

We can proceed now to establish the relation between $\varepsilon_{\mu\tau}$ and Z_H^{32} in the case where the neutrino source is given by the process $\pi \rightarrow \mu \nu$. This decay is mediated by tree level W^\pm and H^\pm exchanges. The relevant effective Lagrangian describing this process is

$$\frac{4G_F}{\sqrt{2}} V_{ud} (\bar{u} \gamma_\mu P_L d) (\bar{\mu} \gamma^\mu P_L \nu_\mu) + V_{ud} \left(\frac{y_d Z_H^{32*}}{m_{H^\pm}^2} \right) (\bar{u} P_R d) (\bar{\mu} P_L \nu_\tau), \quad (60)$$

where $P_{R,L} = (1 \pm \gamma_5)/2$ and y_d is the down quark Yukawa coupling. Since the π meson is a pseudoscalar, its decay amplitude can be induced only by the axial-vector part of the W^\pm coupling and the pseudoscalar part of the H^\pm coupling. Then, once we implement the PCAC relations

$$\langle 0 | \bar{u} \gamma_\mu \gamma_5 d | \pi^- \rangle = i f_\pi p_\pi^\mu, \quad \langle 0 | \bar{u} \gamma_5 d | \pi^- \rangle = -i f_\pi \frac{m_\pi^2}{m_d + m_u}, \quad (61)$$

we get the amplitudes

$$\mathcal{M}_{\pi \rightarrow \mu \nu_\mu}^W = \frac{G_F}{\sqrt{2}} V_{ud} f_\pi m_\mu \bar{\mu} (1 - \gamma_5) \nu_\mu, \quad (62)$$

$$\mathcal{M}_{\pi \rightarrow \mu \nu_\tau}^H = -\frac{V_{ud} f_\pi}{4} \left(\frac{y_d Z_H^{32*}}{m_{H^\pm}^2} \right) \frac{m_\pi^2}{m_d + m_u} \bar{\mu} (1 - \gamma_5) \nu_\tau. \quad (63)$$

We observe that the SM amplitude depends on the lepton mass because of the helicity suppression, in contrast to the charged Higgs amplitude which does not suffer from this suppression. Yet, many NP models predict the Yukawa couplings to be proportional to the fermion masses. Effectively then, in such models both the W -boson and charged Higgs amplitudes have the same lepton mass dependence.

Finally, recalling that the produced neutrino state is $|\nu^s\rangle = |\nu_\mu\rangle + \epsilon_{\mu\tau}^s |\nu_\tau\rangle$, we identify

$$\varepsilon_{\mu\tau}^\pi \approx (\epsilon_{\mu\tau}^{s*})^\pi = -\frac{\sqrt{2}}{4G_F} \left(\frac{m_\pi^2}{m_d + m_u} \right) \left(\frac{y_d Z_H^{32}}{m_\mu m_{H^\pm}^2} \right). \quad (64)$$

In the case of $K \rightarrow \mu \nu$, the relevant parameter $\varepsilon_{\mu\tau}^K$ can be simply obtained from $\varepsilon_{\mu\tau}^\pi$ through the replacement $(m_\pi, y_d, m_d) \rightarrow (m_K, y_s, m_s)$ and we get

$$\frac{\varepsilon_{\mu\tau}^\pi}{\varepsilon_{\mu\tau}^K} \simeq \frac{m_\pi^2}{m_K^2} \frac{m_s + m_u}{m_d + m_u} \frac{y_d}{y_s} \sim \frac{1}{20}. \quad (65)$$

It has yet to be seen which process $\pi \rightarrow \mu \nu$ or $K \rightarrow \mu \nu$ may represent the best probe of this scenario when combining the more intense neutrino flux obtainable from $\pi \rightarrow \mu \nu$ with the higher NP sensitivity of $K \rightarrow \mu \nu$.

IV. ONE LOOP NSIS AND SUPERSYMMETRY

In this section, we apply the model-independent formalism developed in the previous section to the case of the R-parity conserving MSSM with new sources of LFV in the soft sector. We will analyse first loop-induced NSIs from the $V - A$ charged current, passing then to NSIs from the charged scalar current induced by the heavy Higgs sector of the MSSM.

A. $V - A$ charged current

In the MSSM NSIs may be induced by the $V - A$ charged current through W -penguin as well as box contributions. The former effect arises only after the EWSB and therefore is suppressed by M_W^2/M_{NP}^2 . In particular, within the MSSM, there are three possible sources of $SU(2)$ breaking:

- i) the D -terms,
- ii) the left-right mixing terms, and
- iii) the neutralino/chargino mixing terms.

The latter comes from dimension six four-fermion operators and is also suppressed by M_W^2/M_{NP}^2 .

The full analytical calculation relevant for NSIs in the MSSM, upon which our numerical analysis is based on, is reported in the appendix. In the following, instead, we prefer to discuss general properties within an illustrative toy model which is a particular limit of the MSSM that greatly simplifies the calculation but still retains the most relevant features. We use standard MSSM notation (for a review see for example, Ref. [36]) and we consider only the lepton sector.

In our toy model, we decouple the Higgsinos, \tilde{H} , by taking a large μ -parameter and we decouple the Winos, \tilde{W} , by taking their soft mass term, M_2 , to be very large. We also assume left-right mixing between the sleptons to be negligible. Finally, we take the EW vacuum expectation value, v_{EW} , small compared to the soft mass term of the Bino, M_1 , and the soft term for the left-handed sleptons, M_L . That is, we consider a model where

$$\frac{A}{M_L}, \frac{m_\tau \mu \tan \beta}{M_L^2} \ll 1, \quad v_{EW} \ll M_1 \sim M_L \ll \mu \sim M_2. \quad (66)$$

where A stands for the trilinear soft terms. In this model there is only one neutralino, the Bino. In practice, the model looks supersymmetric only with respect to $U(1)_Y$. The EWSB can be treated as a perturbation. Note that, in our toy model, only the $SU(2)$ breaking source i) is at work. Later on, we will also discuss the impact on NSIs of sources ii) and iii), which are expected in a more realistic model.

The relevant interactions are the lepton-Bino-slepton vertex and the slepton-slepton- W vertex, that are given by

$$\mathcal{L}_{NSI} = -\sqrt{2}ig'q_Y\bar{\chi}^0 \left(\tilde{\nu}_k^* U_{\tilde{\nu}}^{ki} \nu_i + \tilde{\ell}_k^* U_{\tilde{\ell}}^{ki} \ell_i \right) - \sqrt{2}ig \left(W^\mu \partial_\mu \tilde{\nu}_k \tilde{\ell}_k^* \right) + h.c. \quad (67)$$

where $q_Y = -1/2$ is the hypercharge of the left-handed leptons. $U_{\tilde{\nu}}^{ki}$ ($U_{\tilde{\ell}}^{ki}$) is the unitary matrix that diagonalizes the sneutrino (charged slepton) mass matrix. In our model, the soft



FIG. 1: 1-loop contributions to the lepton self-energies (left) and to the vertex (right). The virtual particles running in the loop are sleptons and a neutralino.

terms are $SU(2)_L \times U(1)_Y$ symmetric and, since there are no left-right mixings, $U_{\bar{\nu}} = U_{\bar{\ell}}$. Thus, from this point on we use $U_{\bar{\ell}}$ for both terms.

According to Eq. (31), all we need to calculate are the loops in Fig. 1 and extract η^ν , η^ℓ and η^W . In our calculations we use a naive UV cutoff, that is, we perform the k^2 integral up to Λ^2 . We further introduce an unphysical mass parameter μ to make the arguments of all logarithms dimensionless. Effectively, this is equivalent to introducing a renormalization scale. As a check on our calculation we see that the final physical results, that is, ε , is independent of these two unphysical parameters.

We first calculate the wave function corrections. The only diagrams contributing to η^ν at one loop are through the exchange of the Bino and sneutrinos. The result is

$$\eta_{ji}^\nu = g'^2 q_Y^2 \sum_k U_{\bar{\ell}}^{kj*} U_{\bar{\ell}}^{ki} \mathcal{I}_k^\nu, \quad (68)$$

where

$$\mathcal{I}_k^\nu = \frac{1}{16\pi^2} \left\{ \log \frac{\Lambda^2}{\mu^2} + F_k^\nu + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) \right\}, \quad (69)$$

$$F_k^\nu = \left[-\log \left(\frac{m_{\tilde{\nu}_k}^2}{\mu^2} \right) + \frac{m_{\chi_0}^4}{(m_{\tilde{\nu}_k}^2 - m_{\chi_0}^2)^2} \log \left(\frac{m_{\tilde{\nu}_k}^2}{m_{\chi_0}^2} \right) - \frac{m_{\tilde{\nu}_k}^2 + m_{\chi_0}^2}{2(m_{\tilde{\nu}_k}^2 - m_{\chi_0}^2)} \right]. \quad (70)$$

The calculation of the wave function correction to the left-handed charged leptons η_{ji}^ℓ proceeds in a completely analogous way, the only difference being that now we have sleptons, instead of sneutrinos, running in the loop. Therefore, η_{ji}^ℓ reads

$$\eta_{ji}^\ell = g'^2 q_Y^2 \sum_k U_{\bar{\ell}}^{kj*} U_{\bar{\ell}}^{ki} \mathcal{I}_k^\ell, \quad (71)$$

where \mathcal{I}_k^ℓ is simply obtained from \mathcal{I}_k^ν by the replacement $m_{\tilde{\nu}_k}^2 \rightarrow m_{\tilde{\ell}_k}^2$.

Next, we calculate the one loop correction to the W vertex, η^W . The result is

$$\eta_{ij}^W = g'^2 q_Y^2 \sum_k U_{\bar{\ell}}^{kj*} U_{\bar{\ell}}^{ki} \mathcal{I}_k^W, \quad (72)$$

where

$$\mathcal{I}_k^W = \frac{1}{16\pi^2} \left\{ \log \frac{\Lambda^2}{\mu^2} + F_k^W + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) \right\}, \quad (73)$$

$$F_k^W = \frac{1}{(m_{\tilde{\nu}_k}^2 - m_{\tilde{\ell}_k}^2)(m_{\tilde{\nu}_k}^2 - m_{\chi^0}^2)(m_{\tilde{\ell}_k}^2 - m_{\chi^0}^2)} \left[m_{\tilde{\nu}_k}^4 (m_{\chi^0}^2 - m_{\tilde{\ell}_k}^2) \log\left(\frac{m_{\tilde{\nu}_k}^2}{\mu^2}\right) \right. \\ \left. + m_{\tilde{\ell}_k}^4 (m_{\tilde{\nu}_k}^2 - m_{\chi^0}^2) \log\left(\frac{m_{\tilde{\ell}_k}^2}{\mu^2}\right) + m_{\chi^0}^4 (m_{\tilde{\ell}_k}^2 - m_{\tilde{\nu}_k}^2) \log\left(\frac{m_{\chi^0}^2}{\mu^2}\right) \right]. \quad (74)$$

In order to obtain the final result we use Eq. (31) with Eqs. (68), (71), and (72). We get

$$\begin{aligned} \varepsilon &= (\eta^{W\dagger} + \eta^W - \eta^\ell - \eta^\nu) = g'^2 q_Y^2 \sum_k U_{\tilde{\ell}}^{kj*} U_{\tilde{\ell}}^{ki} (2\mathcal{I}_k^W - \mathcal{I}_k^\ell - \mathcal{I}_k^\nu) \\ &= g'^2 q_Y^2 \sum_k U_{\tilde{\ell}}^{kj*} U_{\tilde{\ell}}^{ki} (2F_k^W - F_k^\ell - F_k^\nu), \end{aligned} \quad (75)$$

where we neglected $\mathcal{O}(\Lambda^{-2})$ effects.

We are now in a position to check the finiteness of the physical amplitude. Note that η^ν , η^ℓ , and η^W contain a log-divergence. $SU(2)_L$ gauge symmetry constraints the coefficients of these two divergences to be equal to each other. We can see that this is indeed the case by direct inspection. ε in Eq. (75) depends only on the functions F^a that are independent of Λ . We can also check that the result is independent of μ , as it should be. For this note that the sum, $2F_k^W - F_k^\ell - F_k^\nu$, is independent of μ . While the above results are automatically achieved by each off-diagonal term contributing to ε , as a result of the GIM-mechanism, their validity for the diagonal components represents a check of the correctness of the calculation.

Another important check is to make sure that in the limit of no EWSB, there is no effect induced by the kinetic term because $SU(2)_L$ gauge symmetry makes it aligned with the W -interaction. When sending $v_{EW} \rightarrow 0$, the charged sleptons become degenerate with the sneutrinos, $m_{\tilde{l}_k}^2 = m_{\tilde{\nu}_k}^2$. In this limit, using (75) we learn that the relevant sum is proportional to the identity

$$\varepsilon_{\alpha\beta} \propto \left(\eta_{\alpha\beta}^W + \eta_{\alpha\beta}^{W\dagger} - \eta_{\alpha\beta}^\nu - \eta_{\alpha\beta}^\ell \right) \Big|_{v_{EW}=0} \propto U_{\tilde{\ell}}^\dagger U_{\tilde{\ell}} = \delta_{\alpha\beta}. \quad (76)$$

We learn that no flavor changing amplitude is induced thanks to the unitarity of $U_{\tilde{\ell}}$.

The fact that the effect vanishes for $v_{EW} = 0$ can be used to get an approximate formula. We can define the presumably small parameter

$$a_k \equiv \left(\frac{m_{\tilde{\nu}_k}^2 - m_{\tilde{\ell}_k}^2}{m_{\tilde{\ell}_k}^2 + m_{\tilde{\nu}_k}^2} \right), \quad (77)$$

that vanishes for $v_{EW} = 0$, and expand in a_k to the leading order

$$\varepsilon_{\alpha\beta} = \left(\eta^W + \eta^{W\dagger} - \eta^\nu - \eta^\ell \right)_{\alpha\neq\beta} = \frac{g'^2 q_Y^2}{16\pi^2} U_{\tilde{\ell}}^{k\alpha*} U_{\tilde{\ell}}^{k\beta} \sum_k [a_k^2 G_k + \mathcal{O}(a_k^3)], \quad (78)$$

where G_k is a function of SUSY masses which does not vanish in the limit of $a_k \rightarrow 0$. The dominant splitting between left handed sneutrinos and sleptons originates from the D -terms and is flavor universal

$$(m_{\tilde{\nu}_\alpha}^2 - m_{\tilde{\ell}_\alpha}^2) = m_Z^2 \cos^2 \theta_W \cos(2\beta) . \quad (79)$$

As a result, in this toy model, $\varepsilon_{\mu\tau}$ can be estimated as

$$\varepsilon_{\mu\tau} \sim \frac{\alpha_Y}{4\pi} \cos^4 \theta_W \cos^2(2\beta) \left(\frac{m_Z^2}{\text{Max}[m_{\chi_0}^2, m_{\tilde{\ell}}^2]} \right)^2 \delta_{\mu\tau}^L \lesssim 10^{-6} \delta_{\mu\tau}^L . \quad (80)$$

where we have defined the mass-insertion parameters $\delta_{ij}^L = (M_L^2)_{ij} / \sqrt{(M_L^2)_{ii}(M_L^2)_{jj}}$, as usual. Such values are well below the expected experimental resolutions even for $\delta_{\mu\tau}^L \sim 1$.

We discuss now the main differences arising in the cases where the $SU(2)$ breaking sources ii) and iii) are switched on. In the case ii), the leading effect for $\varepsilon_{\mu\tau}$ reads

$$\varepsilon_{\mu\tau} \sim \frac{\alpha_Y}{4\pi} \frac{m_\tau^2 |A_\tau - \mu \tan \beta|^2}{m_{\chi_0}^2 m_{\tilde{\ell}}^2} \delta_{\mu\tau}^L , \quad (81)$$

where we picked up a double left-right mixing term for the third slepton generation. In principle, $\varepsilon_{\mu\tau}$ could reach values even slightly above 10^{-4} for $m_\tau \mu \tan \beta / m_{\tilde{\ell}}^2 \sim 1$; in practice the constraint from $\tau \rightarrow \mu\gamma$ implies that $\varepsilon_{\mu\tau} < 10^{-5}$.

Finally, in the case iii) we get

$$\varepsilon_{\mu\tau} \sim \frac{\alpha_2}{4\pi} |Z_\pm^{12}|^2 \delta_{\mu\tau}^L , \quad (82)$$

where Z_\pm^{12} are the mixing angles of the chargino mass matrix which read

$$Z_+^{12} \approx \frac{v_u M_2 + v_d \mu}{M_2^2 - \mu^2} \quad Z_-^{12} \approx \frac{v_d M_2 + v_u \mu}{M_2^2 - \mu^2} , \quad (83)$$

where $\tan \beta = v_u / v_d$. We have explicitly checked that also in this case $\varepsilon_{\mu\tau} < 10^{-5}$ after imposing the constraint from $\tau \rightarrow \mu\gamma$.

We discuss now the box induced NSIs. These effects survive even in the limit where all the $SU(2)$ breaking sources discussed above are negligible. In particular, it turns out that the largest effects arise for light sleptons/Winos and heavy Higgsino/Bino. The latter condition is necessary to suppress $\text{BR}(\tau \rightarrow \mu\gamma)$. As a result, it turns out that

$$\varepsilon_{\mu\tau}^{box} \sim \frac{\alpha_2}{4\pi} \frac{m_Z^2}{\text{Max}[M_2^2, m_{\tilde{\ell}}^2]} \delta_{\mu\tau}^L \lesssim 10^{-3} \delta_{\mu\tau}^L . \quad (84)$$

As we will show in the numerical analysis, the box contribution provides the dominant effect to $\varepsilon_{\mu\tau}$ that can reach experimentally interesting levels while being still compatible with the current bound on $\text{BR}(\tau \rightarrow \mu\gamma)$. In fact, in the most favorable situation where $M_2 \sim m_{\tilde{\ell}} \ll \mu \sim M_1$, one can find that

$$|\varepsilon_{\mu\tau}^{box}| \approx 10^{-3} \sqrt{\frac{\text{BR}(\tau \rightarrow \mu\gamma)}{10^{-7}}} , \quad (85)$$

as we will confirm numerically.

B. Scalar charged current

In the MSSM, Higgs mediated LFV effects are generated at the loop level, e.g. see Fig. 2. In fact, given a source of non-holomorphic couplings, and LFV among the sleptons, Higgs-mediated LFV is unavoidable [37].

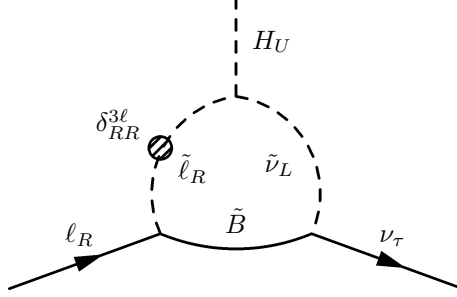


FIG. 2: A contribution to the effective $\bar{\nu}_\tau \ell_R H^+$ coupling.

Starting from the model-independent parameterization for the flavor violating couplings of a charged Higgs with leptons, eq. (55), we specialize now to a SUSY scenario assuming R-parity conservation to avoid tree level flavor changing effects.

Analysing the full expressions of such couplings (reported in the Appendix), we find that the field rotations δL_m and δR_m induce the dominant contributions for the effective Lagrangian of eq. (55). In fact, one can show that they are parametrically enhanced by a $\tan \beta$ factor $\delta L_m, \delta R_m \sim [\alpha_2/4\pi] \times \tan \beta$ compared to $\eta^{\ell,\nu} \sim \alpha_2/4\pi$ and $\eta^H \sim y_\ell \times [\alpha_2/4\pi]$.

Since the effects we are going to discuss can be relevant only if $\tan \beta \gg 1$, it turns out that

$$\mathcal{L}_{\text{eff}}^{H^+} \simeq \bar{\nu}_{jL} [y_\ell^\circ - \delta L_m y_\ell^\circ + y_\ell^\circ \delta R_m]^{ji} \ell_{iR} H^+ + \text{H.c.} \quad (86)$$

Retaining only the dominant $\tan \beta$ enhanced contributions in the corrections to the lepton mass matrix, one has that $\delta m_\ell \simeq \eta_m^\ell$ and therefore

$$(\delta m_\ell)_{ij} \simeq m_{\ell_i}^\circ \epsilon t_\beta \delta_{ij} + \epsilon_R t_\beta \delta_R^{ij} m_{\ell_j}^\circ + m_{\ell_i}^\circ \epsilon_L t_\beta \delta_L^{ij}, \quad (87)$$

where $\epsilon, \epsilon_{L,R}$ are loop factors of order $\alpha_2/4\pi$ which depend on SUSY mass ratios and $t_\beta \equiv \tan \beta$. Therefore, the rotation matrices can be determined explicitly from eqs. (51) and (52) and they read

$$\delta L_m^{3i} \simeq \frac{\epsilon_L t_\beta}{(1 + \epsilon t_\beta)} \delta_L^{3i}, \quad \delta R_m^{3i} \simeq \frac{\epsilon_R t_\beta}{(1 + \epsilon t_\beta)} \delta_R^{3i} + 2 \frac{m_{\ell_i}}{m_\tau} \frac{\epsilon_L t_\beta}{(1 + \epsilon t_\beta)} \delta_L^{3i}. \quad (88)$$

Finally, in the basis where $\nu_L \rightarrow L_m \nu_L$, the effective Lagrangian for the H^\pm couplings with leptons reads

$$\mathcal{L}_{\text{eff}}^{H^+} \simeq \frac{g_2}{\sqrt{2} M_W} \frac{t_\beta}{1 + \epsilon t_\beta} \bar{\nu}_{jL} [m_{\ell_i} \delta^{ji} + m_{\ell_i} t_\beta \Delta_L^{ji} + m_{\ell_j} t_\beta \Delta_R^{ji}] \ell_{iR} H^+ + \text{H.c.} \quad (89)$$

where we have defined $\Delta_{L(R)}^{ji} \equiv \epsilon_{L(R)} \delta_{L(R)}^{ji} / (1 + \epsilon t_\beta)$.

An inspection of the above effective Lagrangian reveals that: 1) since the Yukawa operator is of dimension four, the quantities $\Delta_{L,R}^{ji}$ depend only on ratios of soft SUSY masses, hence avoiding SUSY decoupling. Yet, the NP effects induced in physical observables will decouple with the charged Higgs mass; 2) the loop induced flavor violating couplings are enhanced by an extra t_β factor compared to the tree level flavor conserving couplings. Therefore the loop suppression can be partially compensated if $\Delta_{L(R)}^{ji} t_\beta \sim 1$; and 3) the flavor violating couplings $H^+ \bar{\ell} \nu_\tau$ (with $\ell = e, \mu$) exhibit a Yukawa enhancement factor m_τ/m_ℓ compared to flavor conserving couplings $H^+ \bar{\ell} \nu_\ell$ when they are induced by Δ_R^{ji} .

Applying the above results to the model-independent parameterization of eq. (64), we find

$$\varepsilon_{\mu\tau}^K \approx (\epsilon_{\mu\tau}^{s*})^K = - \left(\frac{m_K^2}{m_{H^\pm}^2} \right) \left(\Delta_L^{32} + \frac{m_\tau}{m_\mu} \Delta_R^{32} \right) \frac{t_\beta^3}{(1 + \epsilon_q t_\beta)(1 + \epsilon t_\beta)}, \quad (90)$$

where ϵ_q is a non-holomorphic threshold correction stemming from the quark sector typically of order $\epsilon_q \sim 10^{-2}$.

As seen in eq. (65), it turns out that $\varepsilon_{\mu\tau}^\pi / \varepsilon_{\mu\tau}^K \approx 1/20$. We notice that $\varepsilon_{\mu\tau}^\pi$ and $\varepsilon_{\mu\tau}^K$ show an enhanced sensitivity to sources of flavor violation in the right-handed slepton sector thanks to the Yukawa enhancement factor m_τ/m_μ .

In order to quantify the allowed size for $\varepsilon_{\mu\tau}^{K,\pi}$, we have to impose the constraints arising from the charged lepton LFV decays. The most sensitive probe of Higgs mediated effects is generally $\tau \rightarrow \ell_j \eta$ [38] and the corresponding branching ratio reads

$$\frac{Br(\tau \rightarrow \mu \eta)}{Br(\tau \rightarrow \mu \bar{\nu}_\nu \nu_\tau)} \approx 10^{-2} \left(\frac{|\Delta_{32}^L|^2 + |\Delta_R^{32}|^2}{m_A^4} \right) \frac{t_\beta^6}{|1 + \epsilon_q t_\beta|^2 |1 + \epsilon t_\beta|^2}, \quad (91)$$

where m_A is the pseudoscalar mass such that $m_A^2 = m_{H^\pm}^2 - M_W^2$ at tree level. Imposing the experimental constraints from $Br(\tau \rightarrow \mu \eta) \lesssim 10^{-7}$, it turns out that

$$\varepsilon_{\mu\tau}^K \lesssim 10^{-2}, \quad \varepsilon_{\mu\tau}^\pi \lesssim 5 \times 10^{-4}, \quad (92)$$

where the above bounds arise for $|\Delta_{32}^L| \ll |\Delta_R^{32}|$.

Finally, let us mention that Higgs mediated LFV interactions also induce lepton universality breaking effects in $P \rightarrow \ell \nu$ ($\ell = e, \mu$) [39]. However, these effects can only constrain $|\Delta_R^{31}|$ which is unrelated, in general, with the relevant LFV term for NSIs, that is $|\Delta_R^{32}|$.

C. Numerical analysis

In this section, we provide the predictions for the NSI parameter $\varepsilon_{\mu\tau}$ in the framework of the R-parity conserving MSSM with generic LFV soft breaking terms. The allowed values for $\mathcal{Im}(\varepsilon_{\mu\tau})$ are obtained after imposing the following constraints: i) the data on flavor physics

observables; ii) the mass bounds from direct SUSY searches; iii) the requirement of a neutral lightest SUSY particle; iv) the requirement of correct electroweak symmetry breaking and vacuum stability; and v) the constraints from electroweak precision observables.

Concerning NSI effects driven by the charged Higgs exchange, the most stringent bounds come from the data on LFV and B -physics observables. In particular, the processes $B \rightarrow X_s \gamma$, $B \rightarrow \tau \nu$ and $B \rightarrow D \tau \nu$ are known to be the most powerful probes of new charged scalar currents. In principle, also the process $B_{s,d} \rightarrow \mu^+ \mu^-$ shows an enhanced sensitivity to extended Higgs sectors. However, since the loop-induced flavor changing coupling $H \bar{b} s(d)$ (with $H = H^0, A^0$) depends on the details of the soft sector, to be conservative, we do not impose here the (model-dependent) constraint from $\text{BR}(B_{s,d} \rightarrow \mu^+ \mu^-)$.

The bounds from $\text{BR}(B \rightarrow X_s \gamma)$ have been obtained employing the SM prediction at the NNLO of Ref. [40], $\text{BR}(B \rightarrow X_s \gamma; E_\gamma > 1.6 \text{ GeV})^{\text{SM}} = (3.15 \pm 0.23) \times 10^{-4}$, combined with the experimental average [41–43] $\text{BR}(B \rightarrow X_s \gamma; E_\gamma > 1.6 \text{ GeV})^{\text{exp}} = (3.55 \pm 0.24) \times 10^{-4}$. As for the SUSY contributions, we use the calculation of Ref. [44] assuming decoupled gluinos and squarks. For $B \rightarrow \tau \nu$, we use the current world average $\text{BR}(B \rightarrow \tau \nu)_{\text{exp}} = (1.73 \pm 0.35) \times 10^{-4}$ [45], the SM prediction $(1.10 \pm 0.29) \times 10^{-4}$ [46] (see also [47]) and the NP contributions of Ref. [48].

Finally, the NP sensitivity of $B \rightarrow D \tau \nu$ can be better exploited normalizing it to $\text{BR}(B \rightarrow D \tau \nu)/\text{BR}(B \rightarrow D \ell \nu)$ where $\ell = e, \mu$ [49, 50]. We use the world average $(49 \pm 10)\%$ [51] and the theoretical prediction of Ref. [52].

In our numerical analysis we impose all the above constraints at the 2σ C.L..

In Fig. (3) on the left, we show the values attained by $|\text{Im}\varepsilon_{\mu\tau}^K|$, see eq. (90), in the $\tan\beta - M_{H^\pm}$ plane setting the LFV parameter $|\Delta_R^{32}| = 10^{-3}$ (varying Δ_R^{32} , $|\text{Im}\varepsilon_{\mu\tau}^K|$ would rescale according to $|\Delta_R^{32}|/10^{-3}$). The red, green, blue and yellow regions are excluded by the current bounds on $B \rightarrow X_s \gamma$, $B \rightarrow \tau \nu$, and $B \rightarrow D \tau \nu$ and $\tau \rightarrow \mu \eta$, respectively.

As shown by fig. (3), $|\text{Im}\varepsilon_{\mu\tau}^K|$ can vary in the range $(10^{-4}, 10^{-2})$ for $\tan\beta \leq 60$ and $M_{H^\pm} \leq 500 \text{ GeV}$. The corresponding values for $|\text{Im}\varepsilon_{\mu\tau}^\pi|$ can be obtained by $|\text{Im}\varepsilon_{\mu\tau}^\pi|/|\text{Im}\varepsilon_{\mu\tau}^K| \approx 1/20$.

We now discuss the NSIs as induced by the $V - A$ charged current via the one loop exchange of gauginos/sleptons. As discussed in the above section, the dominant effect to $\text{Im}\varepsilon_{\mu\tau}$ arises from the box contributions. In fig. (3) on the right, we show the correlation between $\text{BR}(\tau \rightarrow \mu \gamma)$ vs. $|\text{Im}\varepsilon_{\mu\tau}|$ in the case where the neutrino source is provided by the muon decay $\mu \rightarrow e \nu_\tau \bar{\nu}_e$. We have assumed heavy squarks implying negligible NSIs at the detector level. In this limit also NSIs for the production process $P \rightarrow \mu \nu_\tau$ are suppressed. Moreover, as the largest effects for $\text{Im}\varepsilon_{\mu\tau}$ are obtained for light sleptons/Winos and heavy Higgsino/Bino (to keep under control $\text{BR}(\tau \rightarrow \mu \gamma)$), we employ the following scan over the SUSY input parameters: $M_2, m_{\tilde{\ell}} \leq 1 \text{ TeV}$, $\mu, M_1 > 500 \text{ GeV}$ and $3 < \tan\beta < 10$.

As shown by fig. (3), $|\text{Im}\varepsilon_{\mu\tau}|$ can reach experimentally interesting values $|\text{Im}\varepsilon_{\mu\tau}| \lesssim 3 - 4 \times 10^{-4}$ and this would unambiguously imply a lower bound for $\text{BR}(\tau \rightarrow \mu \gamma)$ quite close to

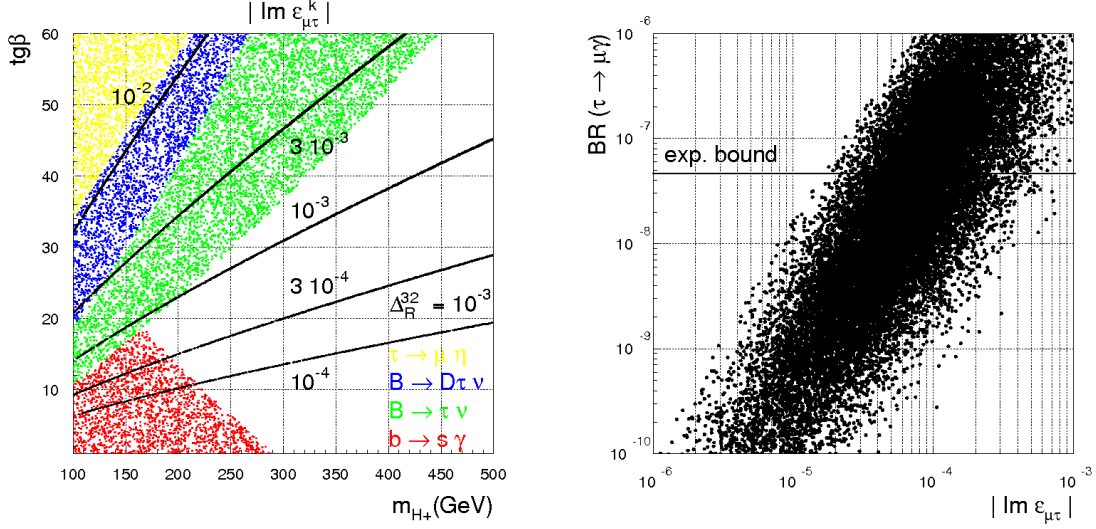


FIG. 3: Left: NSIs in the process $K \rightarrow \ell \nu$ induced by Higgs mediated effects. Right: NSIs in the process $\mu \rightarrow e \nu \bar{\nu}$ induced by W -penguin and gaugino/slepton boxes. See the text for details.

the current bound.

V. DISCUSSION AND CONCLUSION

The idea that neutrino oscillations can probe NSIs is very attractive. In theory such experiments are sensitive to any form of new physics that makes the produced and detected neutrinos non-orthogonal. Such non-orthogonality, parameterized by $\varepsilon_{\alpha\beta}$, may come from new tree level interactions, new heavy neutrinos, or one loop effects that modify the couplings of the W boson to the leptons.

In this work we presented a general framework that allows one to extract in a consistent way the physical ε arising at the loop level either from the $V - A$ or scalar charged currents. We show how ε can be obtained from the various loop amplitudes which include vertex corrections, wave function renormalizations, mass corrections as well as box diagrams.

As an illustrative example, we discussed NSIs in the R-parity conserving MSSM with new LFV sources in the soft sector.

We argued that, in general, the size of one-loop NSIs is quite small, $\varepsilon \approx \mathcal{O}(10^{-3})$. To be observed, such small numbers require very precise measurements of the neutrino appearance probability as a function of L/E . We hope that such measurements will be possible in the next generation of neutrino oscillation experiments.

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VI. APPENDIX

In the following, we provide the full analytical expressions for the self-energies and vertex corrections relevant for NSIs in the R-parity conserving MSSM. For the Feynman rules, we closely follow the notation of Ref. [53].

The lepton self-energies read

$$-(4\pi)^2 (\eta_{\nu L}^\nu)^{IJ} = L_{\nu LC}^{Iki} L_{\nu LC}^{Jki*} B_1(m_{L_k}, m_{C_i}) + L_{\nu \tilde{\nu} N}^{Iki} L_{\nu \tilde{\nu} N}^{Jki*} B_1(m_{\tilde{\nu}_k}, m_{N_i}) \quad (93)$$

$$-(4\pi)^2 (\eta_{VL}^\ell)^{IJ} = L_{eLN}^{Iki} L_{eLN}^{Jki*} B_1(m_{L_k}, m_{N_i}) + L_{e\tilde{\nu} C}^{Iki} L_{e\tilde{\nu} C}^{Jki*} B_1(m_{\tilde{\nu}_k}, m_{C_i}) \quad (94)$$

$$-(4\pi)^2 (\eta_{VR}^\ell)^{IJ} = R_{eLN}^{Iki} R_{eLN}^{Jki*} B_1(m_{L_k}, m_{N_i}) + R_{e\tilde{\nu} C}^{Iki} R_{e\tilde{\nu} C}^{Jki*} B_1(m_{\tilde{\nu}_k}, m_{C_i}) \quad (95)$$

$$(4\pi)^2 (\eta_{mL}^\ell)^{IJ} = -L_{eLN}^{Iki} R_{eLN}^{Jki*} B_0(m_{L_k}, m_{N_i}) - L_{e\tilde{\nu} C}^{Iki} R_{e\tilde{\nu} C}^{Jki*} B_0(m_{\tilde{\nu}_k}, m_{C_i}). \quad (96)$$

The vertex corrections relevant for $W\ell\nu$ are

$$\begin{aligned} (4\pi)^2 (\eta^W)^{IJ} = & \frac{1}{2} L_{\nu \tilde{\nu} N}^{Jkj*} L_{eLN}^{Iij} Z_\nu^{Lk*} Z_L^{Li*} \left[B_0(m_{L_i}, m_{\tilde{\nu}_k}) + \frac{1}{2} + m_{N_j}^2 C_0(m_{N_j}, m_{L_i}, m_{\tilde{\nu}_k}) \right] + \\ & + L_{\nu LC}^{Jki*} L_{eLN}^{Ikj} \left[\sqrt{2} L_{wCN}^{ji} m_{C_i} m_{N_j} C_0(m_{L_k}, m_{C_i}, m_{N_j}) + \right. \\ & \left. - \frac{1}{\sqrt{2}} R_{wCN}^{ji} \left(B_0(m_{C_i}, m_{N_j}) - \frac{1}{2} + m_{L_k}^2 C_0(m_{L_k}, m_{C_i}, m_{N_j}) \right) \right] \\ & + L_{\nu \tilde{\nu} N}^{Jkj*} L_{e\tilde{\nu} C}^{Iki} \left[-\sqrt{2} R_{wCN}^{ji} m_{C_i} m_{N_j} C_0(m_{\tilde{\nu}_k}, m_{C_i}, m_{N_j}) + \right. \\ & \left. + \frac{1}{\sqrt{2}} L_{wCN}^{ji} \left(B_0(m_{C_i}, m_{N_j}) - \frac{1}{2} + m_{\tilde{\nu}_k}^2 C_0(m_{\tilde{\nu}_k}, m_{C_i}, m_{N_j}) \right) \right]. \end{aligned} \quad (97)$$

The vertex corrections relevant for $H\ell\nu$ are

$$\begin{aligned} (4\pi)^2 (\eta^H)^{IJ} = & -V_{\tilde{\nu} LH}^{ml} L_{\nu \tilde{\nu} N}^{Jmn*} R_{eLN}^{Ilm} m_{N_n} C_0(m_{N_n}, m_{\tilde{\nu}_m}, m_{L_l}) \\ & + L_{\nu LC}^{Jnm*} R_{eLN}^{Inl} [L_{NCH}^{lm} C_2(m_{L_n}^2, m_{C_m}^2, m_{N_l}^2) \\ & \quad - R_{NCH}^{lm} m_{C_m} m_{N_l} C_0(m_{L_n}^2, m_{C_m}^2, m_{N_l}^2)] \\ & + L_{\nu \tilde{\nu} N}^{Jnl*} R_{e\tilde{\nu} C}^{Inm} [L_{NCH}^{lm} C_2(m_{\tilde{\nu}_n}^2, m_{N_l}^2, m_{C_m}^2) \\ & \quad - R_{NCH}^{lm} m_{N_l} m_{C_m} C_0(m_{\tilde{\nu}_n}^2, m_{N_l}^2, m_{C_m}^2)]. \end{aligned} \quad (98)$$

The gaugino/slepton box diagrams relevant for the process $\mu \rightarrow e\nu_\tau\bar{\nu}_e$ read

$$\begin{aligned}
-(4\pi)^2\epsilon_{IJ}^{\text{box}} = & L_{\ell LN}^{Ikj} L_{\nu LC}^{Jki*} L_{\nu LC}^{eli} L_{\ell LN}^{elj*} D_2(m_{L_k}, m_{L_l}, m_{C_i}, m_{N_j}) \\
& + L_{\ell\bar{\nu}C}^{IKi} L_{\nu\bar{\nu}N}^{JKj*} L_{\nu\bar{\nu}N}^{eLj} L_{\ell\bar{\nu}C}^{eLi*} D_2(m_{\bar{\nu}_K}, m_{\bar{\nu}_L}, m_{C_i}, m_{N_j}) \\
& + \frac{1}{2} L_{\ell LN}^{Ikj} L_{\nu\bar{\nu}N}^{eKj} L_{\nu LC}^{Jki*} L_{\ell\bar{\nu}C}^{eKi*} m_{C_i} m_{N_j} D_0(m_{L_k}, m_{\bar{\nu}_K}, m_{C_i}, m_{N_j}) \\
& + \frac{1}{2} L_{\ell\bar{\nu}C}^{IKi} L_{\nu LC}^{eki} L_{\nu\bar{\nu}N}^{JKj*} L_{\ell LN}^{ekj*} m_{C_i} m_{N_j} D_0(m_{L_k}, m_{\bar{\nu}_K}, m_{C_i}, m_{N_j}). \quad (99)
\end{aligned}$$

The box diagrams generated by the gaugino/slepton(squark) exchange contributing to the production process $P \rightarrow \mu\nu_\alpha$ ($P = \pi, K$) and to the detection process read

$$\begin{aligned}
-(4\pi)^2\epsilon_{IJ}^{\text{box}} = & L_{\ell LN}^{Jkj*} L_{\nu LC}^{IKi} L_{uDC}^{dli*} L_{dDN}^{dlj} D_2(m_{\tilde{\ell}_k}, m_{\tilde{d}_l}, m_{C_i}, m_{N_j}) \\
& + L_{\ell\bar{\nu}C}^{JKi*} L_{\nu\bar{\nu}N}^{IKj} L_{uUN}^{dLj*} L_{dUC}^{dLi} D_2(m_{\bar{\nu}_K}, m_{\tilde{u}_L}, m_{C_i}, m_{N_j}) \\
& + \frac{1}{2} L_{\ell LN}^{Jkj*} L_{uUN}^{dKj*} L_{\nu LC}^{IKi} L_{dUC}^{dKi} m_{C_i} m_{N_j} D_0(m_{L_k}, m_{\tilde{u}_K}, m_{C_i}, m_{N_j}) \\
& + \frac{1}{2} L_{\ell\bar{\nu}C}^{JKi*} L_{uDC}^{dki*} L_{\nu\bar{\nu}N}^{IKj} L_{dDN}^{dkj} m_{C_i} m_{N_j} D_0(m_{\tilde{d}_k}, m_{\bar{\nu}_K}, m_{C_i}, m_{N_j}). \quad (100)
\end{aligned}$$

The expressions for the loop functions appearing in the above amplitudes read

$$B_0(m_1, m_2) = \frac{1}{\varepsilon} + 1 - \frac{1}{m_1^2 - m_2^2} \left[m_1^2 \log \frac{m_1^2}{\mu^2} - m_2^2 \log \frac{m_2^2}{\mu^2} \right], \quad (101)$$

$$B_1(m_1, m_2) = -\frac{1}{2} \left[\frac{1}{\varepsilon} + 1 - \log \frac{m_2^2}{\mu^2} + \left(\frac{m_1^2}{m_1^2 - m_2^2} \right)^2 \log \frac{m_2^2}{m_1^2} + \frac{1}{2} \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \right], \quad (102)$$

$$C_0(m_1, m_2, m_3) = \frac{1}{m_2^2 - m_3^2} \left[\frac{m_2^2}{m_1^2 - m_2^2} \log \frac{m_2^2}{m_1^2} - \frac{m_3^2}{m_1^2 - m_3^2} \log \frac{m_3^2}{m_1^2} \right], \quad (103)$$

$$C_2(m_1, m_2, m_3) = \frac{1}{\varepsilon} + 1 + \log \frac{m_1^2}{\mu^2} + \frac{m_2^4 \log m_2^2/m_1^2}{(m_1^2 - m_2^2)(m_3^2 - m_2^2)} + \frac{m_3^4 \log m_3^2/m_1^2}{(m_1^2 - m_3^2)(m_2^2 - m_3^2)}, \quad (104)$$

$$\begin{aligned}
D_0(m_1, m_2, m_3, m_4) = & \frac{m_1^2 \log m_1^2}{(m_4^2 - m_1^2)(m_3^2 - m_1^2)(m_2^2 - m_1^2)} \\
& + \{1 \leftrightarrow 2\} + \{1 \leftrightarrow 3\} + \{1 \leftrightarrow 4\}, \quad (105)
\end{aligned}$$

$$\begin{aligned}
D_2(m_1, m_2, m_3, m_4) = & \frac{1}{4} \frac{m_1^4 \log m_1^2}{(m_4^2 - m_1^2)(m_3^2 - m_1^2)(m_2^2 - m_1^2)} \\
& + \{1 \leftrightarrow 2\} + \{1 \leftrightarrow 3\} + \{1 \leftrightarrow 4\}. \quad (106)
\end{aligned}$$

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